Economics 202A
Lecture #1 Outline (version 2.0)

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About This Course

The first part of this course will focus on long-run macro questions, with much of the discussion of short-term fluctuations and the business cycle held off until Economics 202B in the spring.

Thus, we begin by covering various issues in economic growth theory, the basics of consumption and investment theory, the fundamentals asset pricing, and the long-run linkage between money and the price level.

We will depart from this long-run emphasis toward the end of the semester when we discuss informational frictions in financial markets, the dynamic consistency problem in monetary policy, banking instability and (if we get that far), labor markets.

We start off by tackling four issues relating to long-term economic growth:

1. The connections among saving, (exogenous) technology improvements, long-run capital intensity, and long-run per capita income, as recounted by the famous model of Solow (1956).

2. The implications of forward-looking consumers (the Cass-Koopmans-Ramsey model).

3. Issues raised by demographics, including the impact of public debt (primarily Diamond 1965).

4. The implications of viewing growth and technological advance as *endogenous* processes, driven by market incentives (for example, P. Romer 1990).

Throughout, I will feature mathematical "detours" to develop tools and solution methods useful in the application at hand, but also essential to further macroeconomic applications.

Growth Theory: Some Salient Facts

With the Great Depression of the interwar period over, post-World War II economists began to think about how national incomes were determined over the long term by capital accumulation and technological progress. In this regard the first widely influential model was that of Solow (1956). Chapter 1 of David Romer's textbook is required reading for this section of 202A; also, you might glance at Solow's original article on JSTOR.[1]

Throughout our discussion of growth theories we will ask whether they can help us understand the main features of the global economic landscape, so I present some salient facts at the outset.

First of all, and most obviously, there are huge differences in output per capita among countries. (See the following table.) Are these caused by differences in factor endowments? In technology? Something else? This is perhaps the most pressing single question in growth theory – and in development economics.

The time series data on income per capita reflect that growth *rates* of per capita income have differed widely over time. Growth theory also seeks to understand why this is so. A key questions is whether countries that are relatively poor will tend to grow more quickly than their richer neighbors, which might even allow them eventually to catch up. Countries in East Asia like China and Taiwan may be doing this, and indeed, some have recently "graduated" to high-income status. Others (such as many sub-Saharan countries) show no evidence of catch-up: if anything, their relative position has worsened over time. The following chart and graph show that there is no universal tendency toward (unconditional) *convergence* in per capita incomes; some countries seem to be converging, but many others have *di*verged. We want to understand the factors behind this difference.

Assumptions of the Solow Model

First, let's consider *technology.* The fundamental concept in the model is the production function,

$$Y = F(K, AL),$$

where Y is total output (GDP), K is the capital stock, L is the labor force employed, and A represents a technological-knowledge coefficient determining the productivity of labor. An increase in either factor (or in A) raises output. The production function exhibits constant returns to scale, meaning that for any nonnegative constant λ,

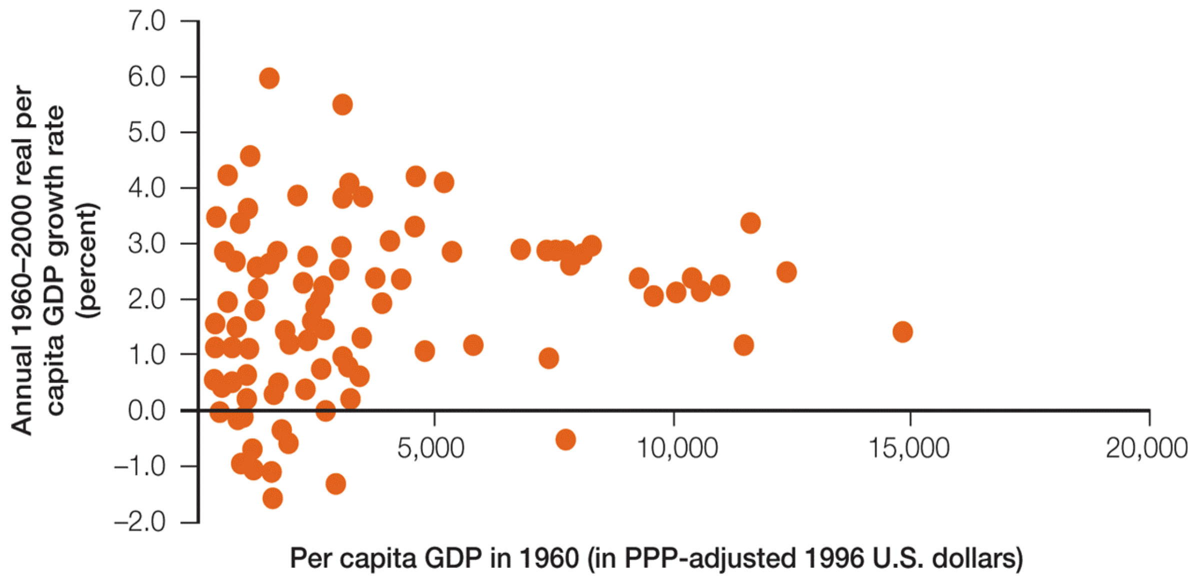
$$F(\lambda K, \lambda AL) = \lambda F(K, AL).$$

[1] A similar model written about the same time was by Trevor Swan, "Economic Growth and Capital Accumulation," *Economic Record* 32 (November 1956): 334-61.

GDP per capita in year 2000 U.S. dollars

Country	*1960*	*2000*	*Average growth (% per year)*
Canada	10,577	26,821	2.4
France	8,605	25,045	2.7
Ireland	5,380	24,948	3.9
Italy	7,103	22,487	2.9
Japan	4,632	23,971	4.2
Spain	4,965	19,536	3.5
Sweden	10,955	25,232	2.1
United Kingdom	10,353	24,666	2.2
United States	13,030	34,365	2.5
Ghana	372	1,392	3.4
Kenya	1,159	1,268	0.2
Nigeria	1,096	1,074	-0.1
Senegal	1,797	1,571	-0.3
Zimbabwe	2,277	3,256	0.9
Argentina	7,859	11,332	0.9
Brazil	2,670	7,194	2.5
Chile	5,022	11,430	2.1
Colombia	2,806	6,080	2.0
Mexico	3,695	8,082	2.0
Paraguay	2,521	4,965	1.7
Peru	3,048	4,205	0.8
Venezuela	5,968	7,323	0.5
China	445	4,002	5.6
Hong Kong	3,264	27,236	5.4
Malaysia	1,829	11,406	4.7
Singapore	4,211	29,434	5.0
South Korea	1,544	15,702	6.0
Taiwan	1,491	19,184	6.6
Thailand	1,086	6,474	4.6

Poor countries have not grown faster:
growth rates relative to per capita GDP in 1960



Source: Penn World Table, Version 6.1

Define the capital stock per effective worker as

$$k \equiv K/AL.$$

Then, define output per effective worker as

$$y \equiv Y/AL = \frac{1}{AL}F(K, AL) = F(k, 1) \equiv f(k).$$

Notice that because $F(K, AL) = ALf(K/AL)$, then by the chain rule,

$$\frac{\partial F(K, AL)}{\partial K} = ALf'(k)\frac{1}{AL} = f'(k),$$

meaning that $f'(k)$ is the marginal product of capital (MPK). This also implies that the MPK depends only on the ratio of capital to effective labor. We assume the concavity property that $f''(k) < 0$ (diminishing returns to capital deepening).

A property of constant returns production functions is that[2]

$$Y = \frac{\partial F(K, AL)}{\partial K}K + \frac{\partial F(K, AL)}{\partial (AL)}AL. \tag{1}$$

Suppose that capital depreciates at rate $\delta > 0$.Let us denote the *rental rate on capital* by

$$r = f'(k) - \delta.$$

The amount a worker earns (the wage) will be the marginal product of an effective labor unit times the productivity A of each worker, $w = A\frac{\partial F(K,AL)}{\partial (AL)}$.[3]

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[2]This is called Euler's Theorm. Proof: Since

$$F(\lambda K, \lambda AL) = \lambda F(K, AL)$$

we may differentiate both sides with respect to λ and evaluate at $\lambda = 1$ to get

$$F(K, AL) = K\frac{\partial F(K, AL)}{\partial K} + AL\frac{\partial F(K, AL)}{\partial (AL)}.$$

[3]If $\tilde{L} \equiv AL$, than the marginal product of L is

$$\frac{\partial F}{\partial \tilde{L}}\frac{\mathrm{d}\tilde{L}}{\mathrm{d}L} = A\frac{\partial F}{\partial \tilde{L}},$$

by the chain rule.

Then eq. (1) implies that

$$Y - \delta K = rK + wL,$$

(*net* national product equals factor incomes), and, dividing by AL, we see that

$$w = A\left[f(k) - kf'(k)\right].$$

A specific constant returns production function that is often used is the Cobb-Douglas form, $F(K, AL) = K^{\alpha}(AL)^{1-\alpha}$, such that $f(k) = k^{\alpha}$ and $f'(k) = \alpha k^{\alpha-1}$. A property of this function is that labor's share of GDP is

$$\frac{wL}{Y} = \frac{A\left[f(k) - kf'(k)\right]L}{Y} = \frac{\left[f(k) - kf'(k)\right]}{y} = \frac{k^{\alpha} - \alpha k^{\alpha-1} \cdot k}{k^{\alpha}} = 1 - \alpha.$$

Since $Y - \delta K = rK + wL$,

$$\frac{(r+\delta)K}{Y} = \alpha.$$

In the Solow model, $r + \delta$ is the *user cost of capital* – the shadow price to a firm of operating its capital (the forgone rental plus depreciation).

An implication of the Cobb-Douglas assumption is that the labor share in *gross* national product Y should remain constant over time. For U.S. data, this assumption is approximately borne out; see *Economic Report of the President,* February 2008, table B-28. In 1960, the ratio of "compensation of employees" to national income (a rough measure of labor's share $1 - \alpha$, and something of an underestimate, because proprietors, too, contribute labor effort) was 0.62; in 1970, it was 0.66; in 1980, it was 0.68; and in 2006, it was 0.64. Thus, a rule of thumb in which the production function for the U.S. economy is $Y = K^{\frac{1}{3}}(AL)^{\frac{2}{3}}$ is not too far off the mark. Indeed, it is argued that similar numbers characterize other economies.[4]

The remaining assumptions of the model concern *dynamics.* The labor force is fully employed and grows at a constant growth rate n (where a dot over a variable denotes its time derivative):

$$\frac{\dot{L}}{L} = n.$$

[4]For a recent discussion, see Douglas Gollin, "Getting Income Shares Right," *Journal of Political Economy* 110 (April 2002): 458-74.

Importantly, technology is also exogenous and grows at the constant rate g:

$$\frac{\dot{A}}{A} = g.$$

Capital depreciates at the proportional rate $\delta > 0$, as we have assumed, but it can be augmented through saving, with one unit of forgone consumption translating into one unit of K (i.e., there are no frictional costs of transforming output into capital goods – whence the price of capital in terms of consumption is always 1). Unlike in the Cass-Koopmans-Ramsey model to be studied next, saving by households is always a fraction $s > 0$ of income, and this saving flows directly into capital formation (we are abstracting from a government sector for now).

Steady State Capital and the Balanced Growth Path

If C is consumption, the change in the capital stock is fresh saving net of depreciation, giving the key differential equation

$$\begin{aligned} \dot{K} &= Y - C - \delta K \\ &= Y - (1-s)Y - \delta K \\ &= sF(K, AL) - \delta K. \end{aligned} \tag{2}$$

Notice that the proportional (or logarithmic) time derivative of $k = K/AL$ is[5]

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - g - n.$$

So from eq. (2) we derive

$$\frac{\dot{k}}{k} = \frac{sF(K, AL) - \delta K}{K} - g - n,$$

or, multiplying through by $k = K/AL$,

$$\dot{k} = sf(k) - (n + g + \delta)k. \tag{3}$$

[5] Observe that, for example,

$$\frac{\mathrm{d} \ln k}{\mathrm{d}t} = \frac{1}{k} \frac{\mathrm{d}k}{\mathrm{d}t}$$

by the chain rule. Because $\ln k \equiv \ln K - \ln A - \ln L$, taking time derivatives of both sides therefore yields the formula that follows.

This is Solow's critical equation.

The next figure shows a central implication of the model: there is a unique level $\overline{k}$ of capital to efficiency-labor such that, once it is attained, the economy remains in a steady state with $K/AL = \overline{k}$ forever. The steady state is defined by

$$\dot{k} = sf(\overline{k}) - (n + g + \delta)\overline{k} = 0,$$

equivalent to

$$sf(\overline{k}) = (n + g + \delta)\overline{k}.$$

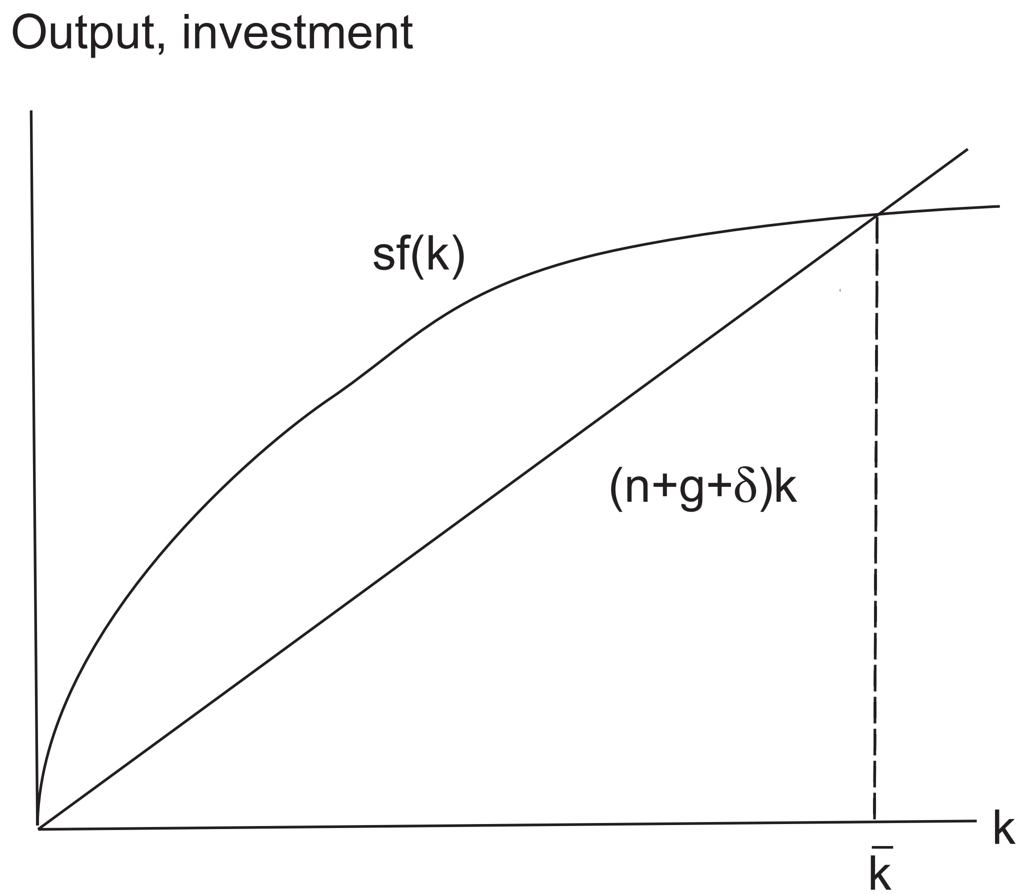
In the steady state, new capital accumulation just offsets the three dynamic forces (n, g, and δ) reducing the ratio of capital to effective labor, so there is no tendency for capital deepening or dilution. Although consumption per efficiency labor unit in steady state is constant and equal to to $\overline{c} = (1-s)f(\overline{k})$, consumption *per worker* grows at the rate of technological advance, g. So does output per worker Y/L, the wage w, and capital per worker K/L. This is a path of balanced growth.

The figure also indicates, importantly, that the economy is *stable*: if k starts off above $\overline{k}$, than $\dot{k} < 0$ and the ratio of capital to AL will fall over time. That ratio will rise over time if the the economy starts off with $k < \overline{k}$. (I am assuming the Inada conditions discussed in David Romer's textbook.)

Implications of the Solow Model

An increase in the saving rate. There will be a transitional period of capital deepening ($\dot{k} > 0$), lasting until the ratio of K to AL settles at a new and higher level of $\overline{k}$. There is a transitional period of higher output growth (why?), but the *long-run growth rate or per capita consumption and income remains constant at* g, which is exogenous.[6] What happens to long-run consumption? This actually could fall if saving drives steady-state capital so high that the economy's higher replacement investment needs exceed the additional output that the extra capital can produce. The issue is whether a higher saving rate drives the economy past the *Golden Rule* capital level defined by Phelps; see his "fable" on the reading list. In this case we say that the balanced growth path is *dynamically inefficient.* You might suspect that this is something that should not happen under laissez-faire, but as we shall

[6] That important feature differentiates this model from the *endogenous growth* models that we will see later. In the latter class of models, various government policies have the potential to affect the long-run growth rate.



Output, investment
sf(k)
(n+g+δ)k
k
k̄

see after we endogenize the saving rate in later models, it cannot be ruled out on theoretical grounds without rather strong assumptions.

An increase in g. In this case $\overline{k}$ must fall, given the saving rate, but the growth rates of output and per capita consumption will obviously be higher along the balanced-growth path. In the model, this is the only change that alters long-run growth.

Convergence. The dynamic stability of the model shows that capital accumulation (hence output growth per efficiency unit of labor) will be positive when k is relatively low and negative when it is relatively high. (Relative to $\bar{k}$, that is.) Since, given A, output per capita varies directly with k, the implication is that poor countries must grow more quickly than rich ones – a prediction seemingly at variance with the data, as we have seen. Of course, it could be that different countries have different As and have long been near their steady states, with no need for transitional dynamic adjustment. (If countries with lower A also have lower $g = \dot{A}/A$, they would tend to fall further and further behind more successful economies in terms of per capita income.) But if A reflects a store of technical knowledge that can flow across boundaries (e.g., via the internet), it is hard to see why different countries would not share the same A, as well as the same long-run rate of growth in labor productivity, g.

Another possiblity is that poor countries are poor because they have low coefficients of saving s and therefore low steady-state levels of capital; perhaps, once again, they have long been near their steady state positions. This idea ignores the ability of (some) countries to borrow from abroad in order to increase their stocks of capital. (The standard Solow model, however, is for a closed economy that must rely entirely on its own savings.)

A related question, therefore, is whether the model can comfortably explain existing cross-country differences in income per capita by differences in capital endowments alone. In 2000, for example, the U.S. was about 30 times richer than Kenya (measured by per capita GDP). If both countries had the same labor efficiency A and identical Cobb-Douglas production functions with $\alpha = 1/3$, then

$$30 \approx \frac{Y_{US}/L_{US}}{Y_{Kenya}/L_{Kenya}} = \left(\frac{K_{US}/L_{US}}{K_{Kenya}/L_{Kenya}}\right)^{1/3},$$

suggesting that

$$\frac{K_{US}/L_{US}}{K_{Kenya}/L_{Kenya}} = 30^3 = 27,000.$$

While Kenya plausibly has less capital per worker than the U.S., the notion that the U.S. is 27,000 richer in this regard seems a bit outlandish. If nothing else, the incentives for capital to move from the U.S. to Kenya would be overwhelming in this case. Forces other than capital scarcity must be at work – for example, cross-country variations in A (due to reasons other than pure disembodied technical knowledge), variations in other productive factors (such as human capital), and the like.

The Solow model is a useful start, but we clearly have more work to do.

The Income Accounting of Hall and Jones

A large literature has pursued the preceding research agenda. One notable and very accessible paper is the one by Bob Hall and Chad Jones in the February 1999 QJE.

The paper makes two important points:

1. Even after adding human capital to the basic Solow model, most of the cross-country variation in output per worker is due *in an accounting sense* to differences, not in factor endowments, but in productivity (the "Solow residual" introduced above).

2. International differences in productivity, as well as in endowments of physical and human capital, can be traced to what Hall and Jones call a country's "social infrastructure": the set of institutions (such as protection of property rights, impartial enforcement of contracts, limits on corruption, excessive taxation, etc.) that ensure that those who undertake productive activities get to keep the bulk of their investments' proceeds.

Hall and Jones focus on the *level* of income rather than its *growth rate* for good reasons that they discuss. You can access their paper through JSTOR or at http://elsa.berkeley.edu/~chad/HallJonesQJE.pdf. In this lecture I will focus on point #1 above while only summarizing the main findings under #2. But please read sections III-VII of the paper too. If you are looking for a nice model of how to apply to macroeconomics what you will be learning in 'metrics, this is an especially clear one.

Hall and Jones start with the Cobb-Douglas production function

$$Y = K^{\alpha} (AH)^{1-\alpha}, \tag{4}$$

where H is defined to be *human capital* rather than raw labor L. Human capital is related to raw labor L, however, through the amount of education a representative worker has acquired. Assuming all workers within a country are alike, the stock or human capital in country i is

$$H_i = \exp\left[\phi(E_i)\right] L_i,$$

where E_i is the number of years a (typical) worker has spent in school, $\phi'(E) > 0$, and $\phi(0) = 0$. The prediction of the preceding equation is that

$$\frac{\mathrm{d}\ln H}{\mathrm{d}E} = \frac{\mathrm{d}H/H}{\mathrm{d}E} = \phi'(E),$$

so that an extra year of schooling raises human capital by $\phi'(E)$ percent. In practice, Hall and Jones compute human capital stocks by using the preceding formulation, together with empirical estimates of the marginal rates of return (in terms of increased lifetime earnings) to various lengths of time in school. For E, they use average 1985 educational attainment of the populations aged 25 and over.

Hall and Jones point out that production function (4) implies

$$Y^{\frac{1}{1-\alpha}} = Y^{\frac{1-\alpha+\alpha}{1-\alpha}} = K^{\frac{\alpha}{1-\alpha}} AH.$$

Dividing this by $Y^{\frac{\alpha}{1-\alpha}}$ and then by raw labor L yields

$$\frac{Y}{L} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{H}{L}\right) A. \tag{5}$$

The expression provides a decomposition of output per worker in terms of the capital intensity of output, human capital per worker, and productivity.

Beware: this decomposition is somewhat mechanical, in that it tells us only how much A matters *given* capital stocks. But the influence of A on per capita output is in fact more powerful than the naive growth accounting exercise would indicate, as a low value of A will, apart from its direct negative effect on output, deter the accumulation of physical and human capital. (Hall and Jones document this later in their paper.)

Hall and Jones calculate human capital as indicated above, and physical capital based on investment since 1960 and an assumed depreciation rate (the widely used but rough and ready "perpetual inventory" method) – see their paper. They also assume $\alpha = \frac{1}{3}$ in all countries. Using these numbers and data on GDP and the labor force, they calculate A for the year 1988 as the residual from (5):

$$A = \frac{Y}{L}\left(\frac{K}{Y}\right)^{\frac{-\alpha}{1-\alpha}}\left(\frac{H}{L}\right)^{-1}.$$

Figure I from their paper (next page) plots their estimate of A against output per worker. As you can see, the positive association is impressive. Of course, this reflects all the channels (direct and indirect) through which A affects output, as discussed a moment ago.

Hall and Jones also report in their Table I the decomposition of Y/L into its proximate determinants, from (5). These numbers normalize the United States to a value of 1.000 and measure all countries' asset endowments and productivities against that benchmark. According to this table, the importance of physical capital intensity in explaining the world income distribution is not that large. This relates to my earlier point regarding Kenya. With $\alpha = \frac{1}{3}$, you need huge K differences to explain the wide global dispersion of incomes, but in practice the differences are not that big, and in Table I, the exponent on K/Y is only $\alpha/(1-\alpha) = \frac{1}{3}/\frac{2}{3} = \frac{1}{2}$. Thus, K cannot play a dominant role. Human capital is somewhat more important. But the big factor seems to be unmeasured productivity, A.

Return to Kenya. In 1988 the U.S. was only 18 times richer than Kenya, not 30 as in 2000 – a fact that illustrates how Kenya's low growth rate compared to the U.S. has hurt it over time (and how quickly growth differences can add up). According to Hall and Jones, in 1988 Kenya had about 45 percent the U.S. level of human capital. However, its ratio of capital to output was $0.747^2 = 0.558$ compared to a normalized ratio of 1 in the U.S., making its capital-labor ratio $K/Y \times Y/L = 0.558 \times 0.056 = 0.031$. We conclude from this that

$$\frac{K_{US}/L_{US}}{K_{Kenya}/L_{Kenya}} = \frac{1}{0.031} = 32.$$

Kenya indeed had much less capital, but its productivity was only 16.5% of U.S. productivity. In contrast, to explain the 1988 income per worker difference by capital alone, we would need to conclude that

$$\frac{K_{US}/L_{US}}{K_{Kenya}/L_{Kenya}} = 18^3 = 5,832.$$

This remains an implausibly high physical capital difference.

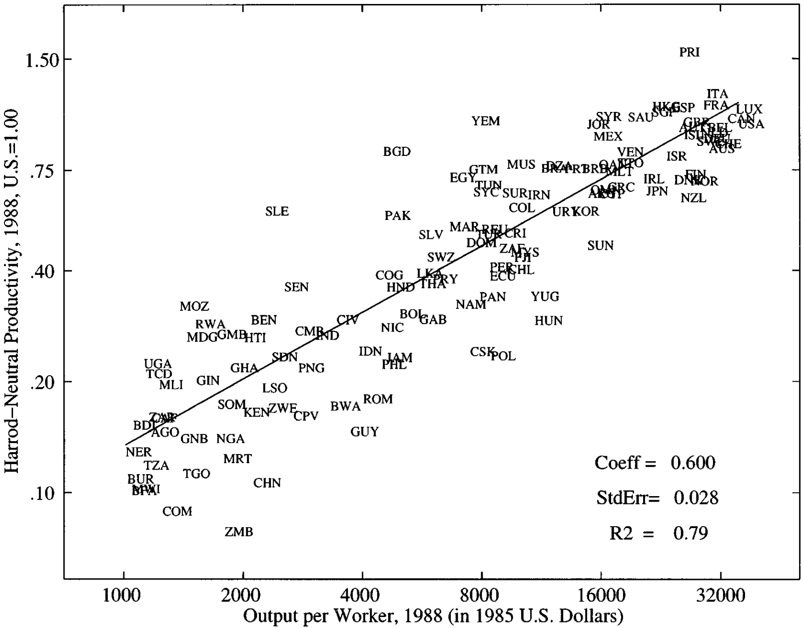


FIGURE I
Productivity and Output per Worker

TABLE I

PRODUCTIVITY CALCULATIONS: RATIOS TO U. S. VALUES

Country	Y/L	Contribution from $(K/Y)^{\alpha/(1-\alpha)}$	Contribution from H/L	Contribution from A
United States	1.000	1.000	1.000	1.000
Canada	0.941	1.002	0.908	1.034
Italy	0.834	1.063	0.650	1.207
West Germany	0.818	1.118	0.802	0.912
France	0.818	1.091	0.666	1.126
United Kingdom	0.727	0.891	0.808	1.011
Hong Kong	0.608	0.741	0.735	1.115
Singapore	0.606	1.031	0.545	1.078
Japan	0.587	1.119	0.797	0.658
Mexico	0.433	0.868	0.538	0.926
Argentina	0.418	0.953	0.676	0.648
U.S.S.R.	0.417	1.231	0.724	0.468
India	0.086	0.709	0.454	0.267
China	0.060	0.891	0.632	0.106
Kenya	0.056	0.747	0.457	0.165
Zaire	0.033	0.499	0.408	0.160
Average, 127 countries:	0.296	0.853	0.565	0.516
Standard deviation:	0.268	0.234	0.168	0.325
Correlation with Y/L (logs)	1.000	0.624	0.798	0.889
Correlation with A (logs)	0.889	0.248	0.522	1.000

The elements of this table are the empirical counterparts to the components of equation (3), all measured as ratios to the U. S. values. That is, the first column of data is the product of the other three columns.

Because the level of residual productivity A is so important in understanding the world income distribution, what explains its level? Hall and Jones construct a measure of "social infrastructure" consisting of two elements:

- An index of the government's provision of protection to property owners of expropriation, either by private parties or by the government itself.
- An index of openness to international trade.

In a cross section of countries, they show that an average of these two factors has a positive effect not only on output per capita and on A itself, but on the appropriately normalized stocks of physical and human capital. For example, in a regression of the form

$$\log Y/L = \alpha + \beta S + \epsilon, \tag{6}$$

where S is the constructed index of social infrastructure, they find an estimate of $\hat{\beta} = 5.143$,with a standard error of 0.508.

In estimating β consistently, they address two main econometric challenges. First, S is likely a noisy proxy for the "true" level of infrastructure, so there is a problem of errors in variables. Second, there is reverse feedback from Y/L to S – for example, richer societies have more resources to devote to protecting property rights – and so S and ϵ are likely positively correlated in the preceding regression equation (simultaneous equations bias).

To deal with these problems, Hall and Jones need to find instrumental variables that influence infrastructure but do not appear in eq. (6). They propose a number of these, some based on geographical factors and colonial antecedents unlikely to be direct determinants of national output today. I refer you to the paper for details, but note that the search for explanations of Y/L based on colonial origins has been a continuing theme in the literature on growth and development.

Economics 202A, Problem Set 1

Maurice Obstfeld

1. *Hicks Meets Solow.* Consider the Solow model, but now with the assumption that

$$Y = AF(K, L),$$

where

$$\frac{\dot{A}}{A} = g.$$

(a) Show what happens if we try to derive a balanced growth path like the one derived in class. (The Solow model features labor-augmenting technical change or Harrod-neutral technical change; the form of technical change shown above is called Hicks neutral.) (b) What can you say in the special case $F(K, L) = K^{\alpha}L^{1-\alpha}$?

2. *Investment Rates in the Transition.* In the Solow model, imagine the economy starts out at some initial capital intensity ratio k_0 that is very close to 0. (a) Show how the investment rate (relative to effective labor supply growth) will change over time (by graphing $\dot{k}$ against time) as $k \to \bar{k}$. (b) At what level of the capital stock is $\dot{k}$ maximized?

3. *The Golden Rule.* (a) Find the level of steady-state capital intensity $\bar{k}$ at which consumption per capita is maximized. (b) What saving rate s^* leads to this golden-rule balanced growth path? (c) Explain this result intuitively.

4. *Solow is So Slow....* Assume a discrete-time Solow model in which $L_{t+1} = (1 + n)L_t$ and $A_{t+1} = (1 + g)A_t$. Define z by

$$(1 + z) \equiv (1 + n)(1 + g).$$

(a) Show that with a Cobb-Douglas production function, the Solow model is summarized by the dynamic equation:

$$k_{t+1} - k_t = \frac{sk_t^{\alpha} - (z + \delta)k_t}{1 + z}.$$

(a) Calculate the value of steady-state capital intensity $\overline{k}$. (b) Define the deviation from the steady-state as $\tilde{k}_t \equiv k_t - \overline{k}$. Show that a first-order Taylor approximation to the preceding equation, valid near $k = \overline{k}$, is

$$\tilde{k}_{t+1} - \tilde{k}_t = \frac{s\alpha\overline{k}^{\alpha-1}\tilde{k}_t - (z+\delta)\tilde{k}_t}{1+z}.$$

(c) Show that another way to write this expression is as the difference equation in $\tilde{k}_t$:

$$\tilde{k}_{t+1} = \left[1 + \frac{(\alpha-1)(z+\delta)}{1+z}\right]\tilde{k}_t.$$

(d) For a given initial value $\tilde{k}_0 = k_0 - \overline{k}$ that is not too big, solve for the approximate value of k_t, capital intensity at time t. (e) Assuming that $\alpha = \frac{1}{3}$ and that n, g, and δ are measured at annual rates as $n = 0.01$, $g = 0.02$, and $\delta = 0.3$, compute the *half-life* (in years) of the distance from steady-state capital intensity – the number of years it takes the distance from the steady state to fall by half.